AFFINE TRANSFORMATION

Lecture 9
5 Dec 2022

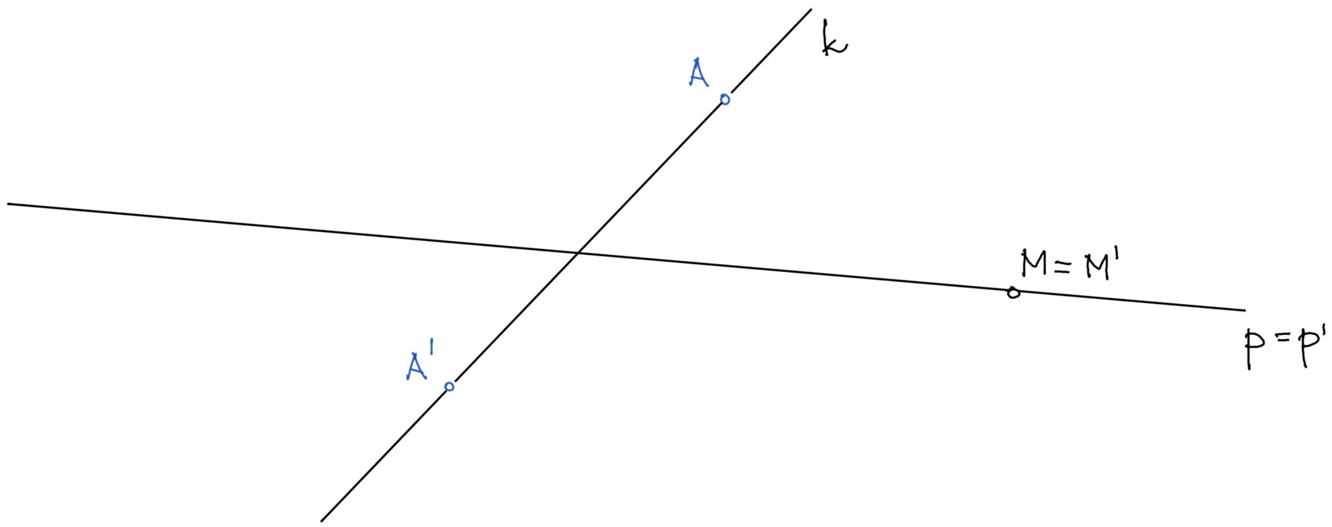
Transformation on a plane, such that

- incidence of points and lines
- colinearity of points
- parallelism of lines
- segment division ratio

} are preserved

Angles are not preserved (in general).

1. Affinity axis and affinity direction.



A, A' – given points

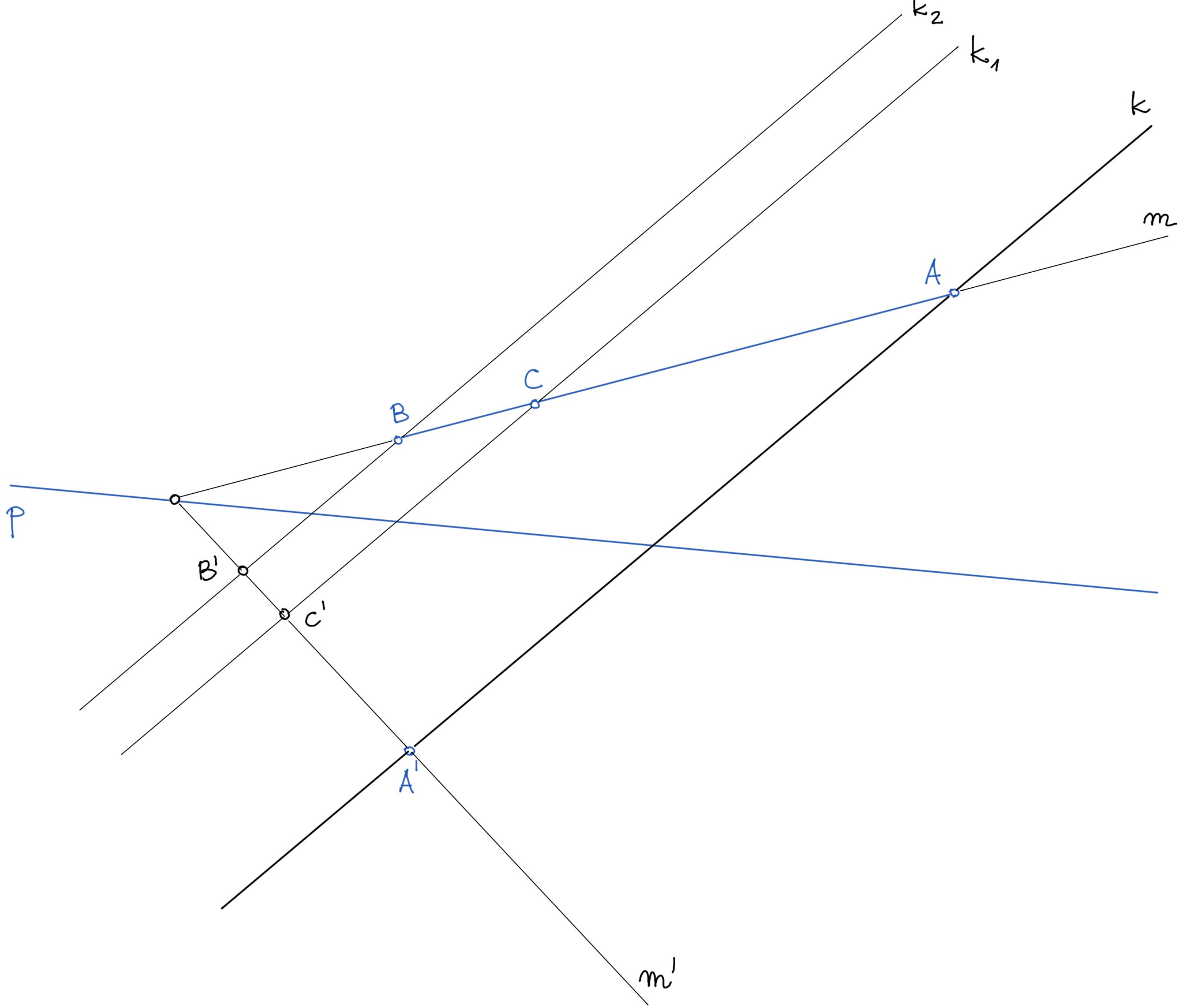
$k(A, A')$ – affinity direction

(A, A') – pair of related points

p – affinity axis

$\{p, (A, A')\}$ – affinity

2. Division of a segment.



Given:

p – affinity axis

(A, A') – related points

AB, $C \in AB$

Problem:

Prove, that

$$\frac{|AB|}{|BC|} = \frac{|A'B'|}{|B'C'|}$$

Solution:

$k_1 \parallel k_2 \parallel k$

$B \in m \Rightarrow B' \in m'$

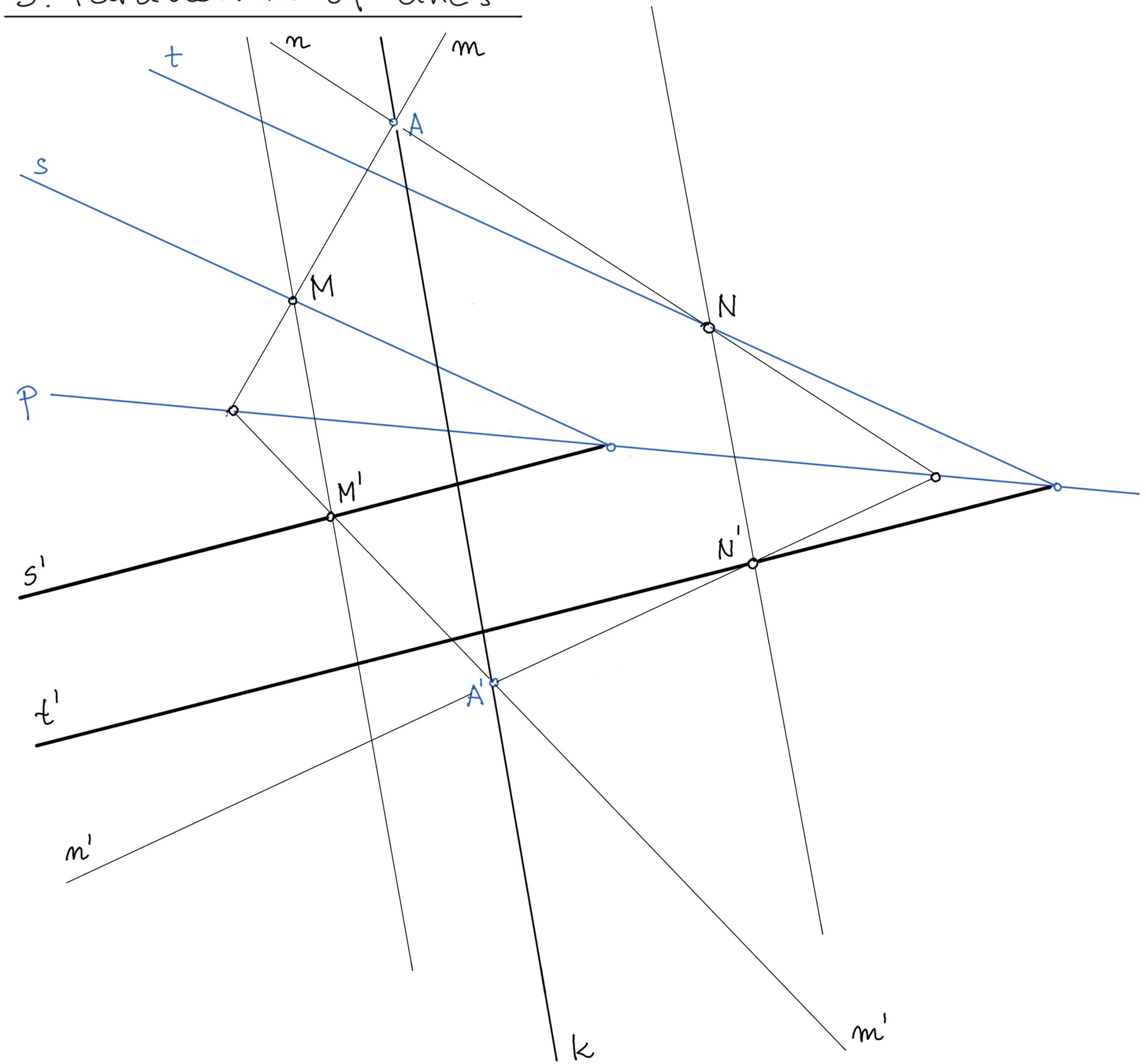
$C \in m \Rightarrow C' \in m'$

The ratio is preserved by the Tales Theorem.

Moreover,

$$\frac{|AC|}{|BC|} = \frac{|A'C'|}{|B'C'|}$$

3. Parallelism of lines



Given:

p – affinity axis

(A, A') – related points

s, t, s ∥ t

Problem:

Find the related lines s', t'.

Justify, that s' ∥ t'.

Solution:

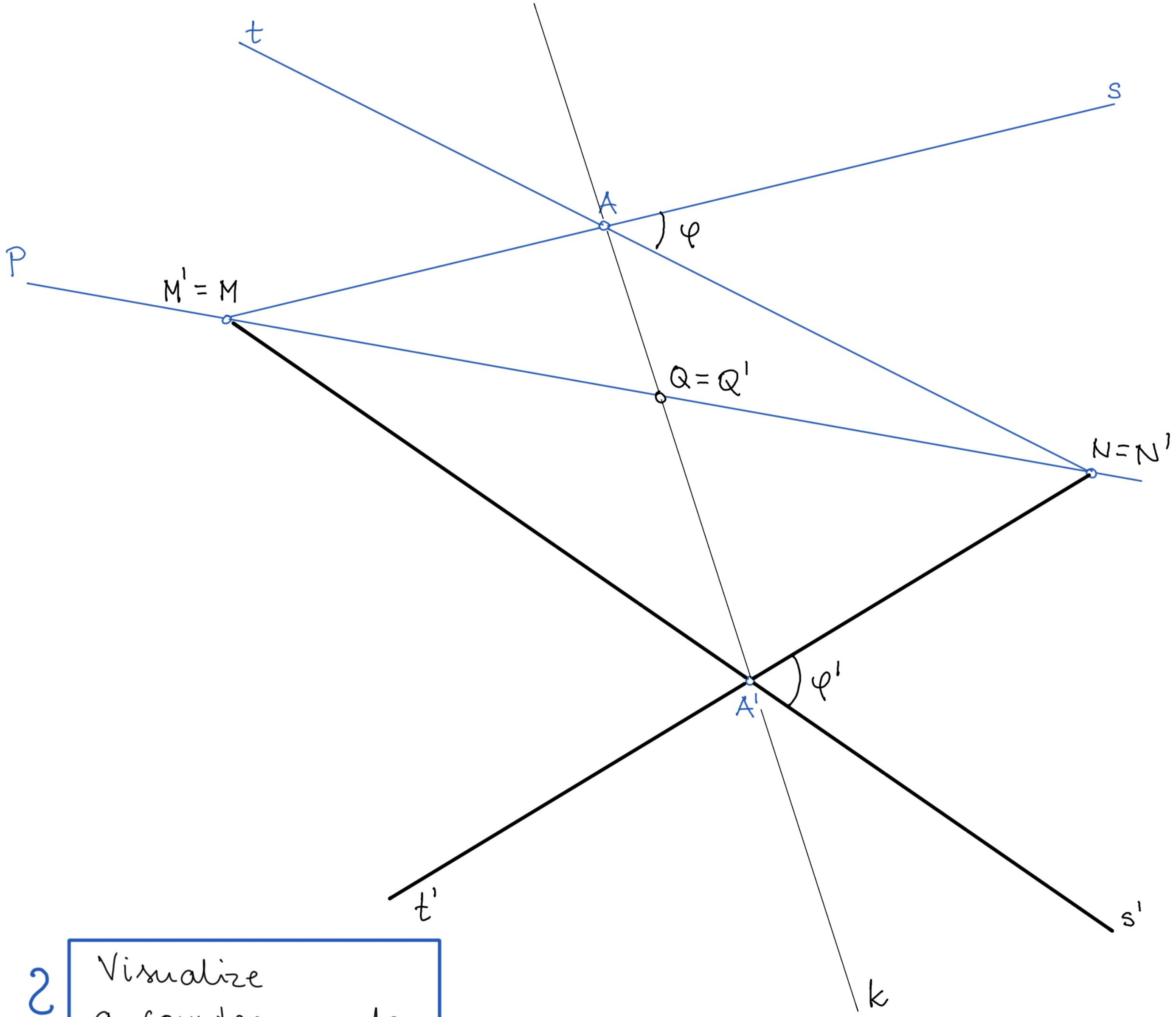
$M \in s$, $N \in t$

m(A, M)

n(A, N)

The parallelism of s', t' follow from the reverse of the Tales Theorem.

4. Angle between two lines.



Given:

p – affinity axis

A, A' – related points

s, t – given lines,

$A \in \{s, t\}$

Problem:

Find s', t' and

$\varphi = \measuredangle(s,t)$

$\varphi' = \measuredangle(s',t')$

Justify, that $\varphi \neq \varphi'$.

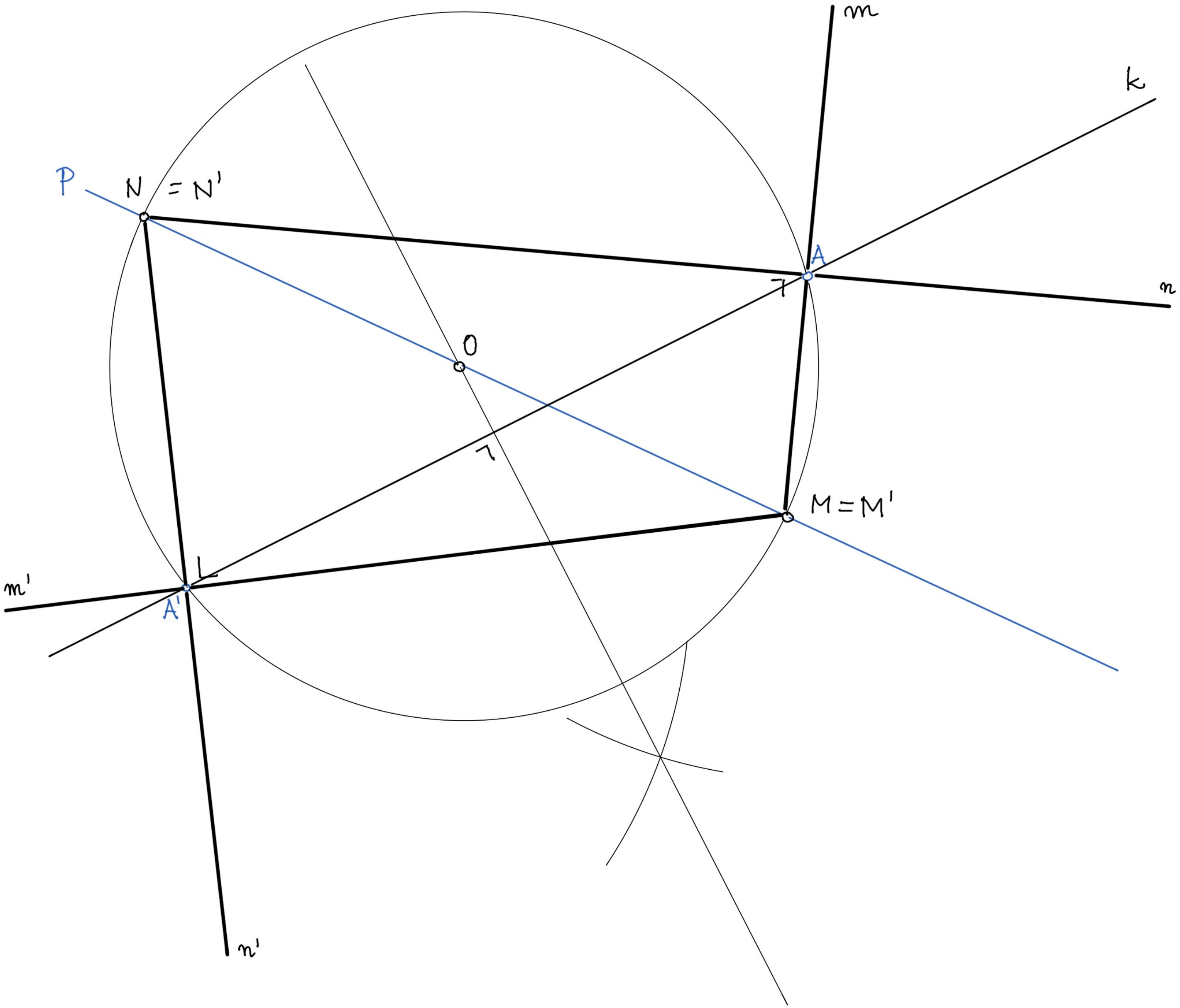
Solution:

Discuss the angles in $\triangle AMN$ and $\triangle A'M'N'$,

or $\triangle ANQ$, AMQ and $\triangle A'N'Q'$, $A'M'Q'$.

? Visualize a counterexample.

5. Principal axes of the affine transformation.



Given:

p – affinity axis

(A, A') – related points

Problem:

Define the principal axes.

$AM \perp AN$

$A'M' \perp A'N'$

m (A, M) } principal
n (A, N) } axes of affinity

$\measuredangle NAM = \measuredangle N'A'M' = 90^\circ$

Problem

Draw a triangle KLM and its affine transformation K'L'M' in a given affinity {p, (A,A')}.
Assume that, ∡KLM = ∡K'L'M' = 90°.

Solution :

- Find (n, m) and (n', m') - principal axes
- Take arbitrary L.
- Let $n_1 \parallel n$ and $m_1 \parallel m$. Then, $n_1' \perp m_1'$
- Take arbitrary $K \in n_1$, $M \in m_1$.
- Find K', M'.