RABATMENT

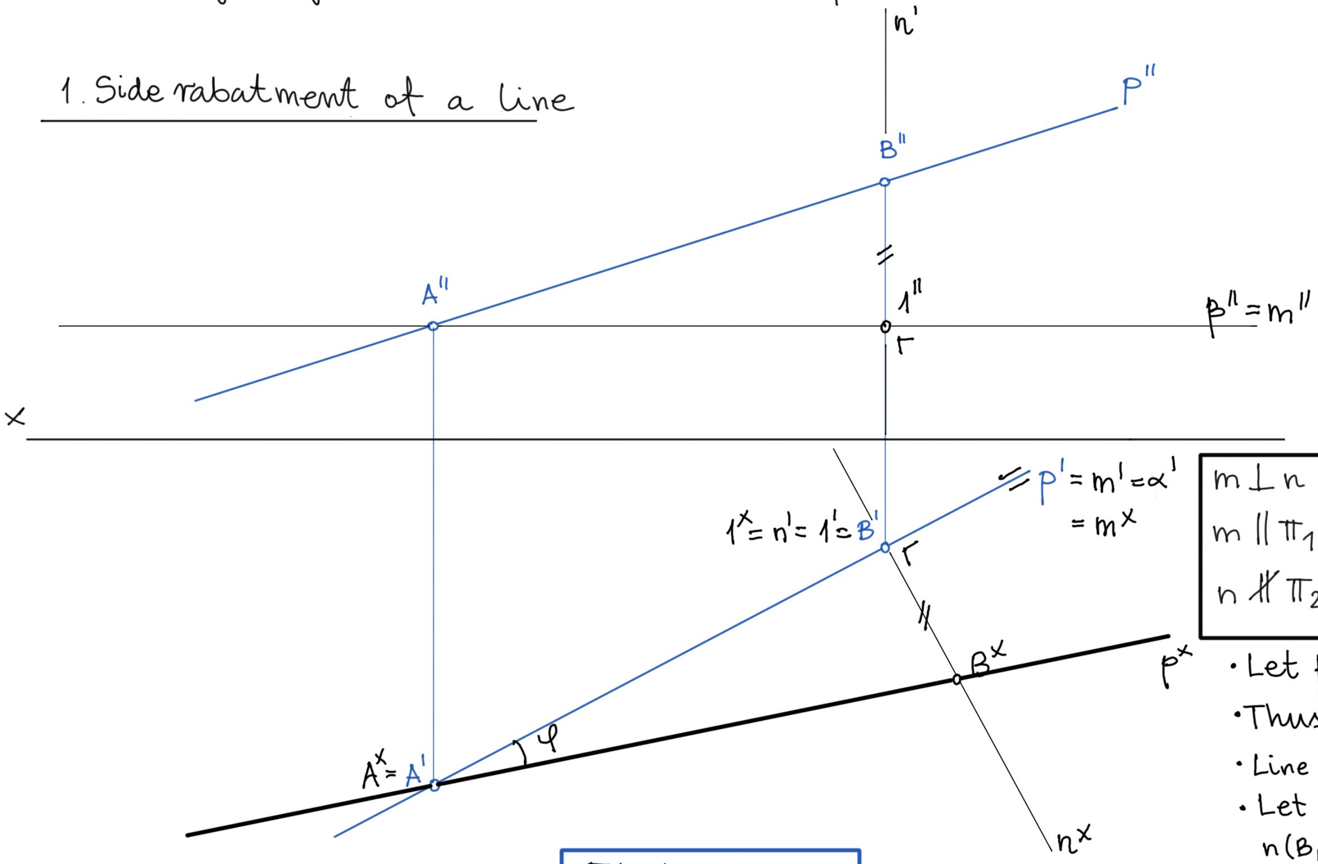
Lecture 4
24 Oct 2022

- rotation about a line parallel to the projection plane to the position parallel to that plane.

$\boxed{\text{Application of the rabatment — measuring distances and angles.}}$

Usually, objects are laid down at a plane $\parallel \Pi_1$.

1. Side rabatment of a line



$\boxed{\text{Find } \psi = \measuredangle(p, \Pi_2)}$?

GIVEN:

$p(A,B)$ – line in general position

PROBLEM:

Find the angle $\varphi = \measuredangle(p, \Pi_1)$

Find the distance $d = |AB|$

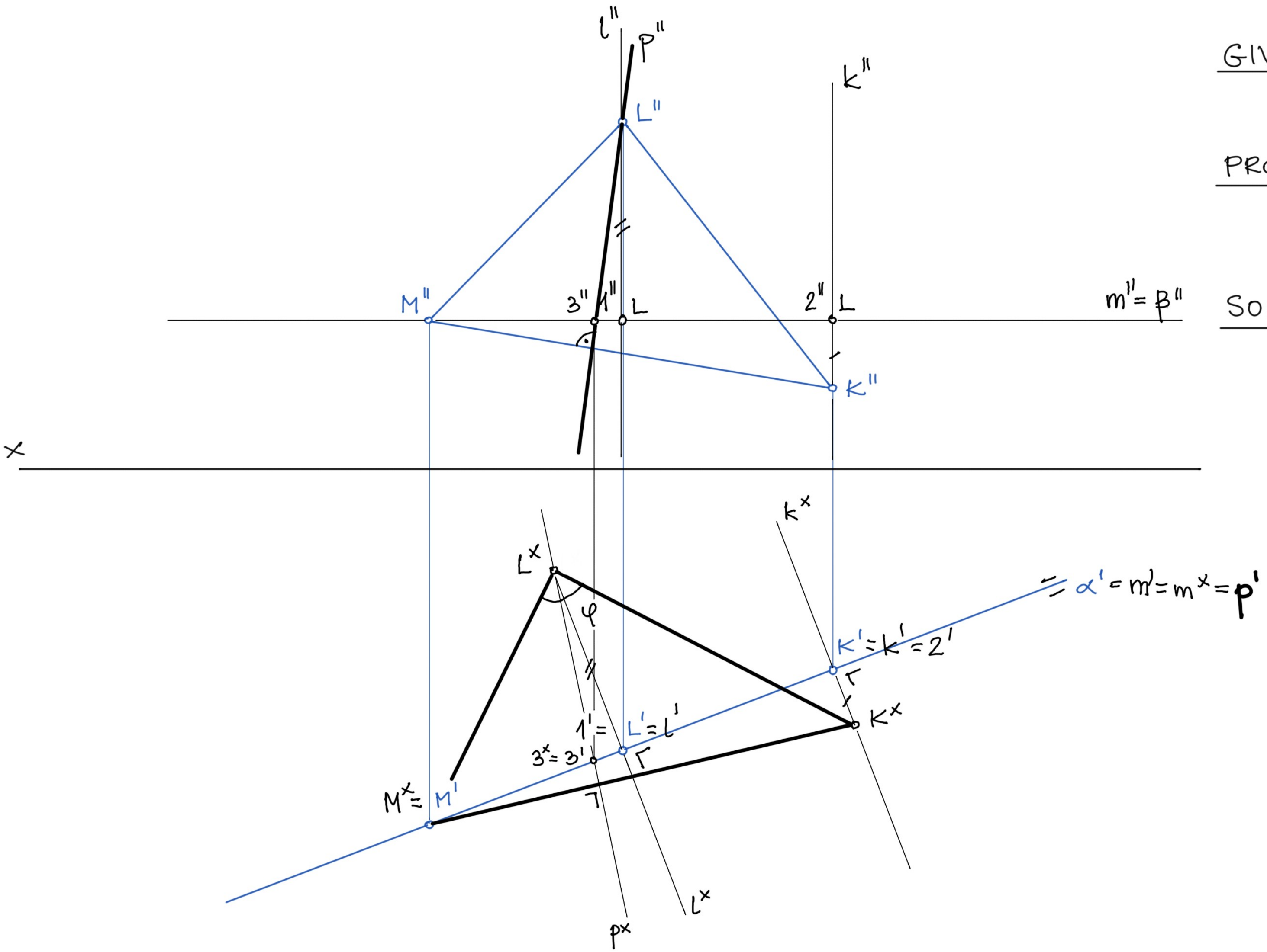
SOLUTION:

- Use the fact: (invariant $1.7^{\perp}$)

$$\boxed{\left.\begin{matrix} m \perp n \\ m \parallel \Pi_1 \\ n \not\parallel \Pi_2 \end{matrix}\right\} \mapsto m' \perp n' ; \quad \left.\begin{matrix} m \perp n \\ m \parallel \Pi_2 \\ n \not\parallel \Pi_1 \end{matrix}\right\} \mapsto m'' \perp n''}$$

- Let $\beta \parallel \Pi_1$ – plane of rabatment, $A = p \cap \beta$.
- Thus, $\measuredangle(p, \Pi_1) = \measuredangle(p, \beta) \overset{df}{=} \measuredangle(p, m)$, $m = \alpha \cap \beta$, $\alpha \perp \beta$.
- Line m is the rotation axis of $\alpha\,(p, m)$
- Let $1 \ni m$ such that $B1 \perp m$. Thus, $B1 \parallel \Pi_2$, $n(B,1) \parallel \Pi_2$, $n'' \perp m''$.
- Hence, $|B1| = |B''1''| \longrightarrow$ we find B^x from $|B''1''| = |B^x1^x|$
- Finally, $\varphi = \measuredangle(p^x, m^x)$, $d = |A^x B^x|$.

2. Rabatment of the projecting plane (side rabatment)



GIVEN:

$\alpha(K, L, M) \perp \pi_1$

PROBLEM:

Find $\varphi = \sphericalangle(KL, LM)$.

Find $p \perp q(K,M)$, $p \ni L$

SOLUTION:

- Use $1.7^{\perp}$
- $\beta \parallel \pi_1$, $M \ni \beta$
- $m \ni M$, $m \ni \beta$, $m \ni \alpha$
- $k \perp \beta$, $l \perp \beta$
- $|1L| = |1''L''| = |1^x L^x|$
- $|2K| = |2''K''| = |2^x K^x|$
- $3 \ni m \rightarrow 3' = 3^x$

? Solve a similar problem, assumming $\Delta \ni \beta \perp \pi_2$

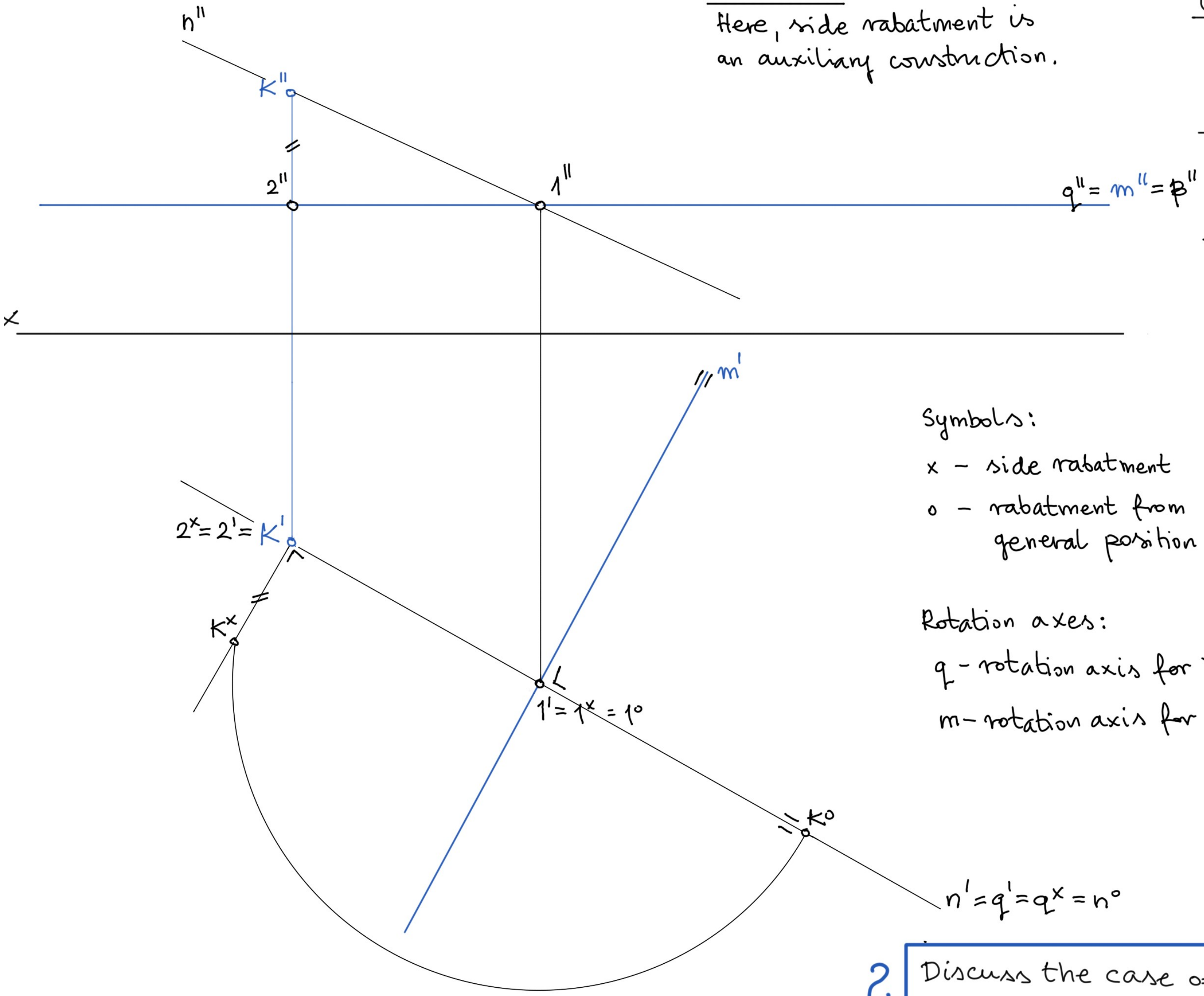
3. Rabatment of a plane in general position.

REMARK:
Here, side rabatment is an auxiliary construction.

GIVEN:
$\alpha(m, K)$, $K \not\in m$, $m \parallel \pi_1$

PROBLEM:
Find $d = |mK|$.
Lay down α.

SOLUTION:
- Use 1.7$^{\perp}$
- $|K'1'| = d' \neq d$!!
- $|K''1''| = d'' \neq d$!!
- Let $\beta \parallel \Pi_1$, $m \in \beta$
- Let $n(1, K) \perp m$, $1 = n \cap m$
- Take γ – auxiliary plane, such that $\gamma \perp \beta$ and $\gamma(K, 1, 2)$, where $2 \in \beta$ and $K2 \perp \beta$.
- Thus, $K2 \parallel \Pi_2$ and $|K2| = |K''2''|$
- Also, $q(1,2) = \gamma \cap \beta$ and $n(1,K) = \gamma \cap \alpha$, $\{q, n\} \perp m$
- Side rabatment of γ – rotation about axis q – allows for finding $d = |1^{x}K^{x}|$.
- For the rabatment of α it is sufficient to find $1^{\circ}K^{\circ} \in n^{\circ}$.



Symbols:
x – side rabatment
o – rabatment from general position

Rotation axes:
q – rotation axis for γ
m – rotation axis for α

? Discuss the case of $\alpha(K, L, M)$

Uniqueness of Monge's projection.

? C. Łapińska (2016) Sec. 2.2 Sec. 3

Fact:

Any point $P \in E^3$ uniquely maps to a pair (P', P'').

$$P \longmapsto (P', P'')$$

Any pair (P', P'') uniquely maps to a point $P \in E^3$.

$$(P', P'') \longmapsto P$$

Hence, $P \longleftrightarrow (P', P'')$.

Remark 1:

Points P', P'' form a pair (P', P'') if they are connected by the correspondence line.

Remark 2:

Statement $m \longmapsto (m', m'')$ for lines is true, but its converse

$(m', m'') \longmapsto m$ not always! – see Sec. 3.2.2 in C. Łapińska (2016)

Corollary:

The unique mapping OBJECT $\leftrightarrow$ PROJECTIONS OF THE OBJECT can be obtained through consideration regarding the points defining that object.

E.g. $m(P,Q) \longleftrightarrow \big(m'(P',Q'),\ m''(P'',Q'')\big)$, where $P \neq Q$.

$\alpha(P,Q,R) \longleftrightarrow \big(\alpha'(P',Q',R'),\ \alpha''(P'',Q'',R'')\big)$, where P, Q, R – not colinear.