Lecture 3
17 Oct 2022

- parallel lines
- perpendicular lines
- planes defined by two lines
- incidence of a point (line) with a plane
- line parallel to a plane, plane parallel to a plane
- piercing of a general plane by a line
- line of intersection of two planes

Self-study problems will be marked by "?"

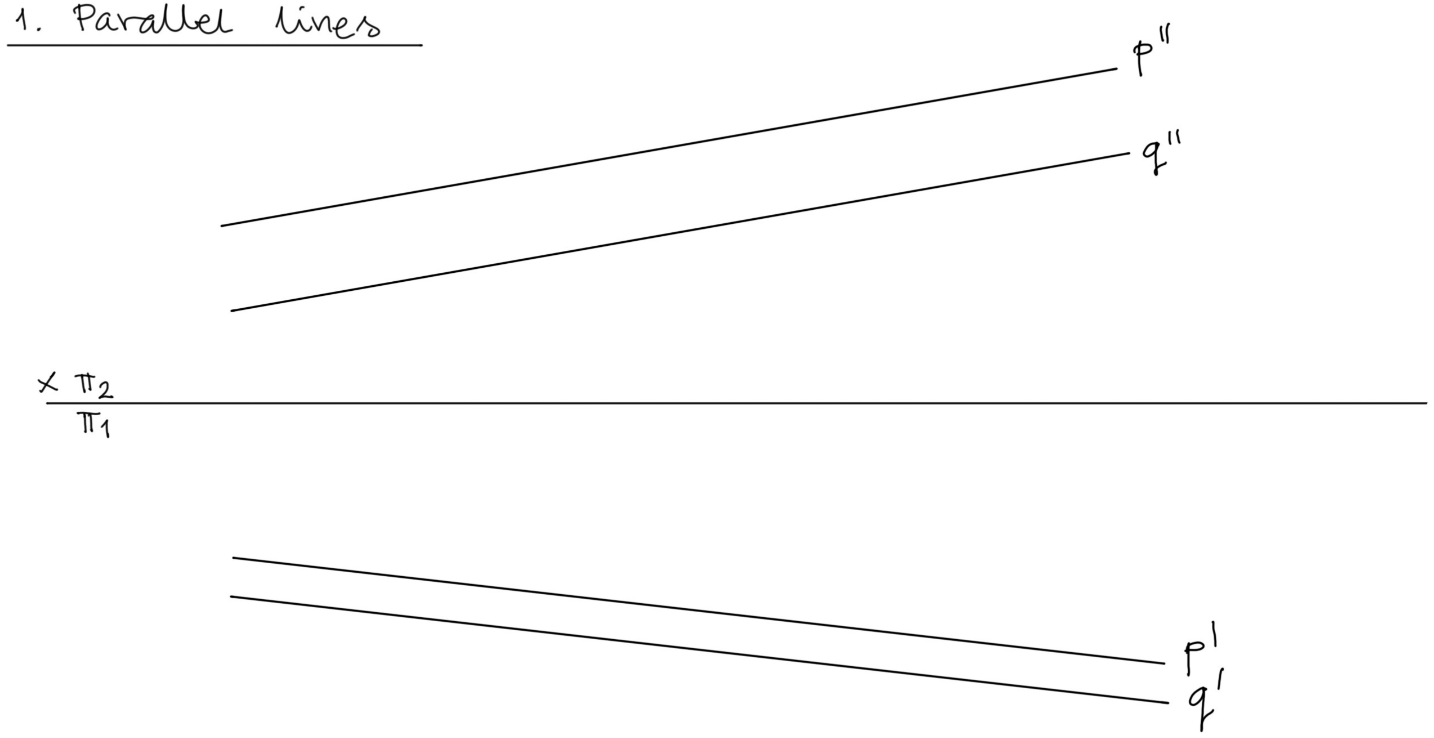
FACT:

$$p \parallel q \mapsto \begin{cases} p' \parallel q' \\ p'' \parallel q'' \end{cases}$$

"$\mapsto$" symbol means "maps to".

Here, two lines that are parallel in space map to two pairs of parallel projections.

1. Parallel lines

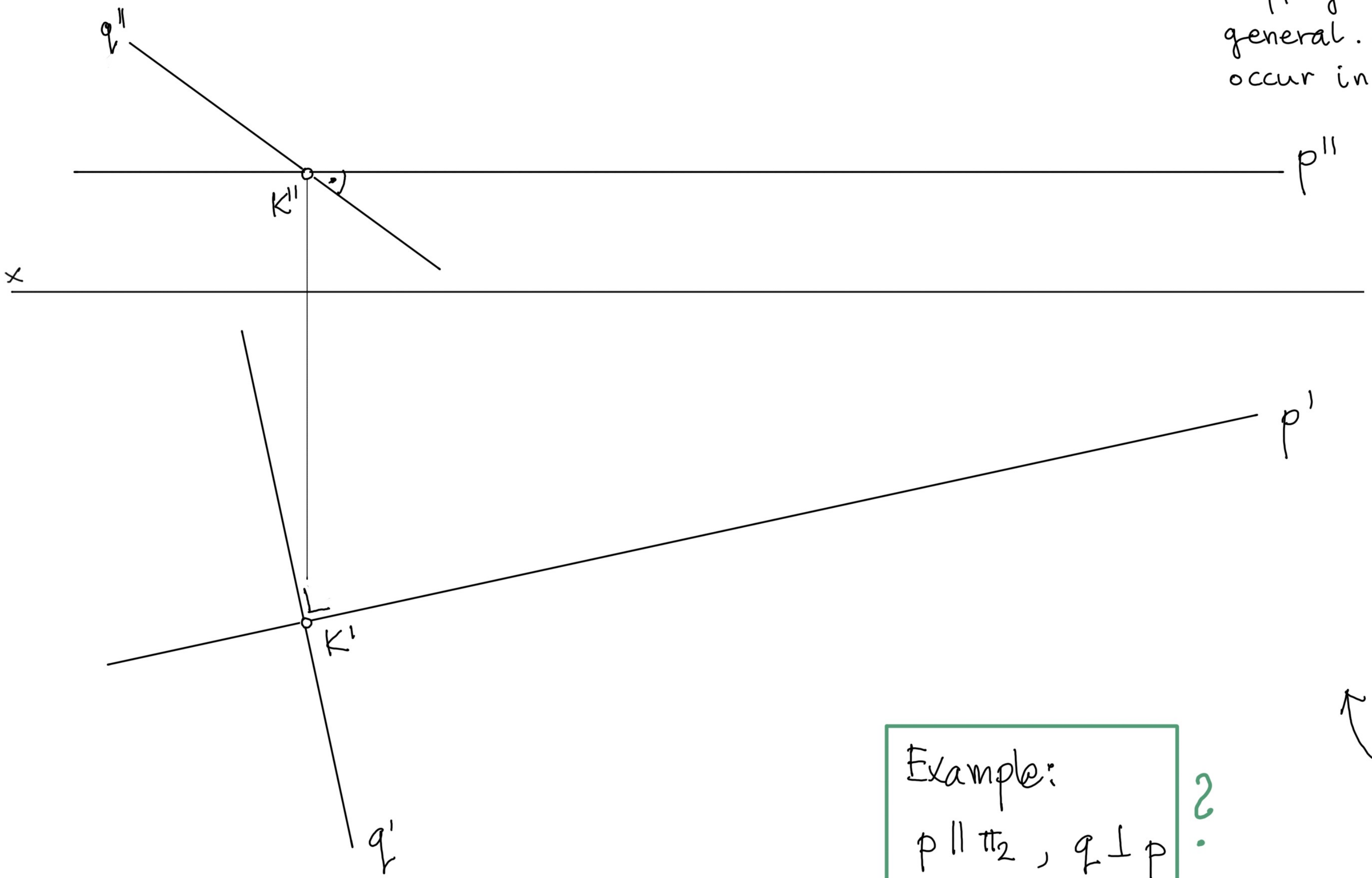


2. Perpendicular lines

FACT:

$$p \perp q \;\not\mapsto\; \begin{cases} p' \perp q' \\ p'' \perp q'' \end{cases}$$

" $\not\mapsto$ " means that the mapping is not true in general. Albeit, it may occur in particular cases.



Special case:

one line (or both) are parallel to projection plane.

Example:

$$\left.\begin{array}{l} p \parallel \pi_1 \\ q \perp p \end{array}\right\} q' \perp p'$$

Example:

$p \parallel \pi_2$, $q \perp p$

?

3. Planes defined by two lines

4. Incidence of a point (line) with a plane

GIVEN: $\alpha(p,q)$, $p \parallel q$, $p \neq q$

PROBLEM: Find a point $K \in \alpha$.
Draw a line $t \in \alpha$.
Draw a line $s \in \alpha$, $s \parallel \pi_1$.
Draw a line $r \perp s$.

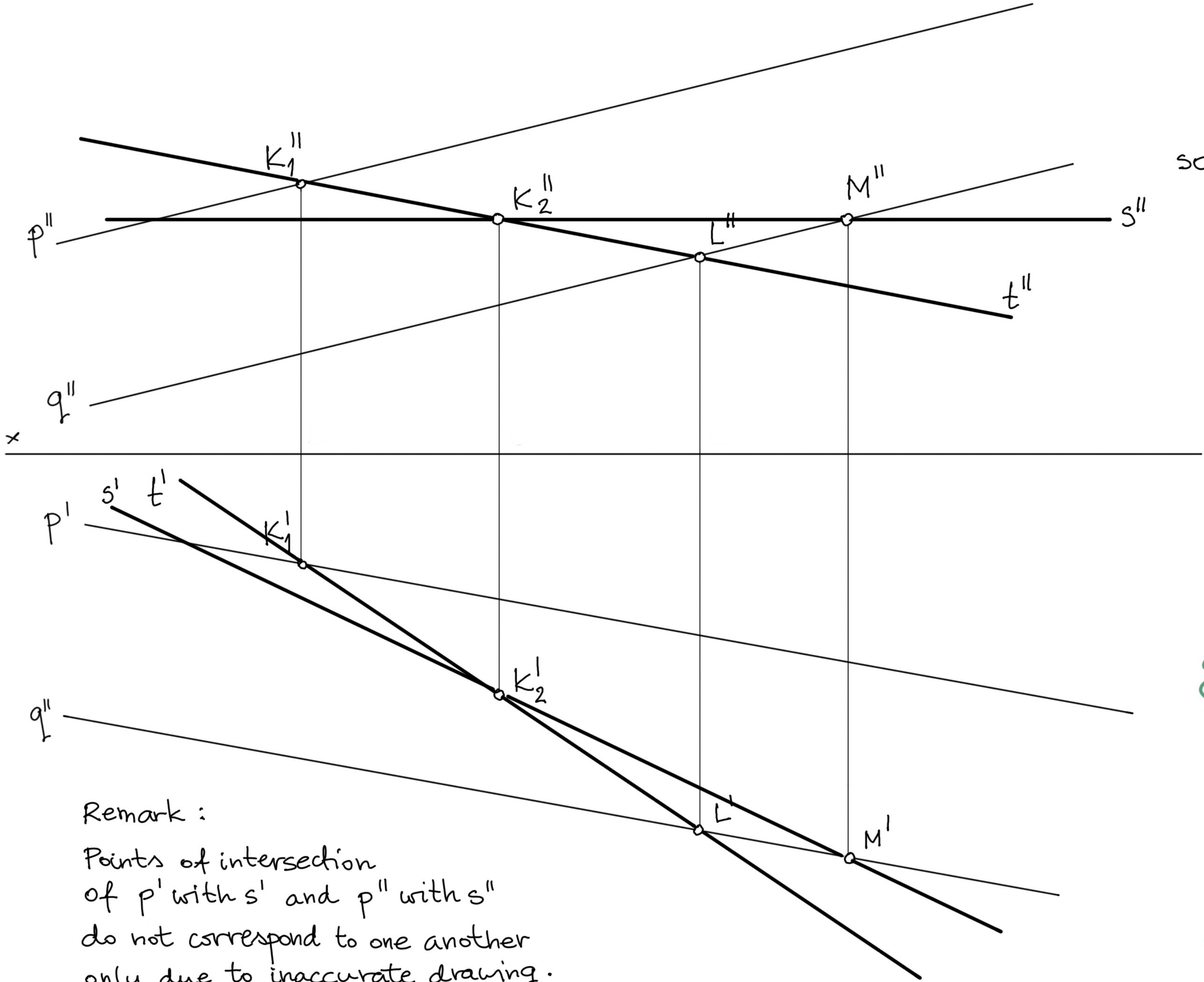
SOLUTION:

$K_1 \in p$ – trivial case

$$\left.\begin{array}{l} L \in q \\ t(K_1,L) \in \alpha \\ K_2 \in t \end{array}\right\} \text{– general case}$$

$s(K_2,M)$

? $\boxed{r \perp s}$



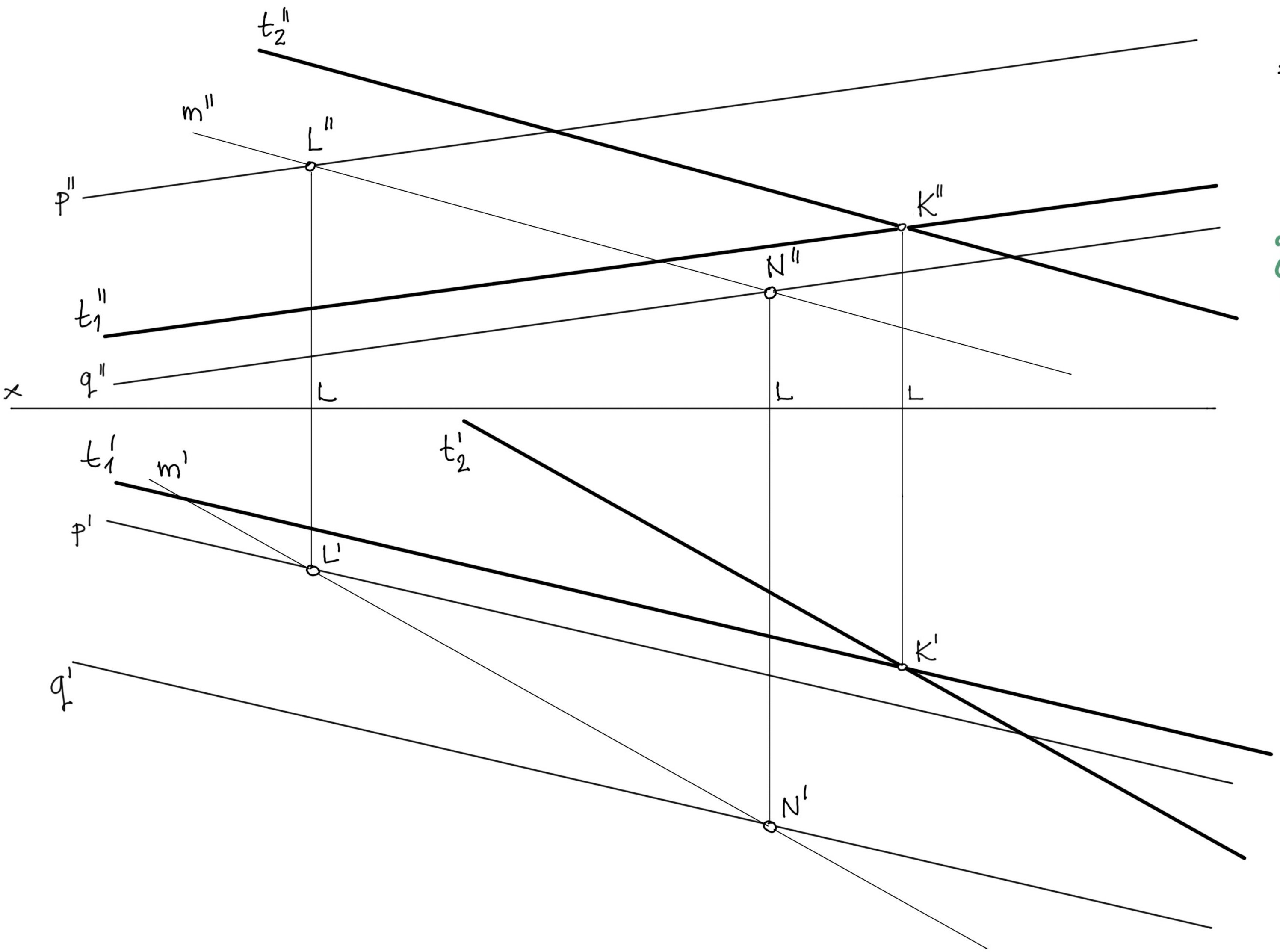
Remark:

Points of intersection of p' with s' and p'' with s'' do not correspond to one another only due to inaccurate drawing.

? Define $\alpha(p,q)$, where p,q – arbitrary with $K = p \cap q$. $p \neq q$
Draw a line $t(K,M)$, where $M \in \alpha$.

? How to draw a point (or line) that is not incident with a given plane?

5. Line (plane) parallel to a given plane



GIVEN:

$\alpha(p,q)$, $p \parallel q$, $p \neq q$ and $K \notin \alpha$.

PROBLEM:

Draw $t \parallel \alpha$ and $\beta \parallel \alpha$ through K.

? How do we check if $K \in \alpha$?

SOLUTION:

$t_1 \parallel \alpha$, $K \in t_1$ - trivial case

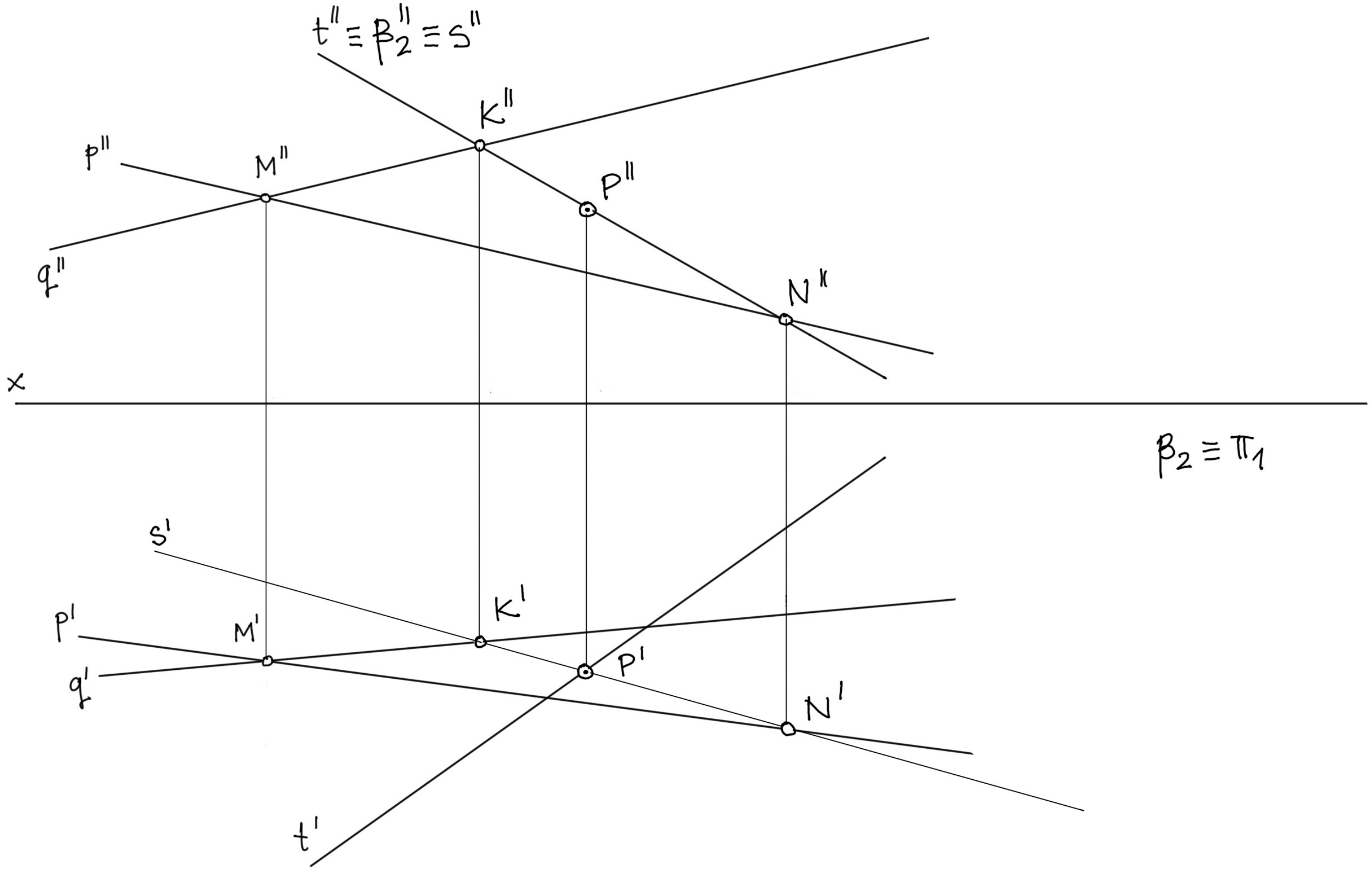
$m \in \alpha$, $t_2 \parallel m$
$L = m \cap p$
$N = m \cap q$
- general case

$K = t_1 \cap t_2$

$\beta(t_1, t_2) \parallel \alpha$

? Let $\alpha(p,q)$, where p,q - arbitrary with $K = p \cap q$. Let also $L \notin \alpha$.
Find M, N such that $\beta(L,M,N) \parallel \alpha$.

6. Piercing of a general plane by a general line



GIVEN: $\alpha(p,q)$ and $t \notin \alpha$.

PROBLEM: Find $K = t \cap \alpha$.

SOLUTION: $\beta \ni t$, $\beta \perp \pi_2$

β - auxiliary plane $\boxed{\beta \perp \pi_1}$?

$K, N \in \beta$

$K = q \cap \beta$

$N = p \cap \beta$

$s \in \beta$ - by assumption

$s(K,N) \in \alpha$

Since also $t \in \beta$, then

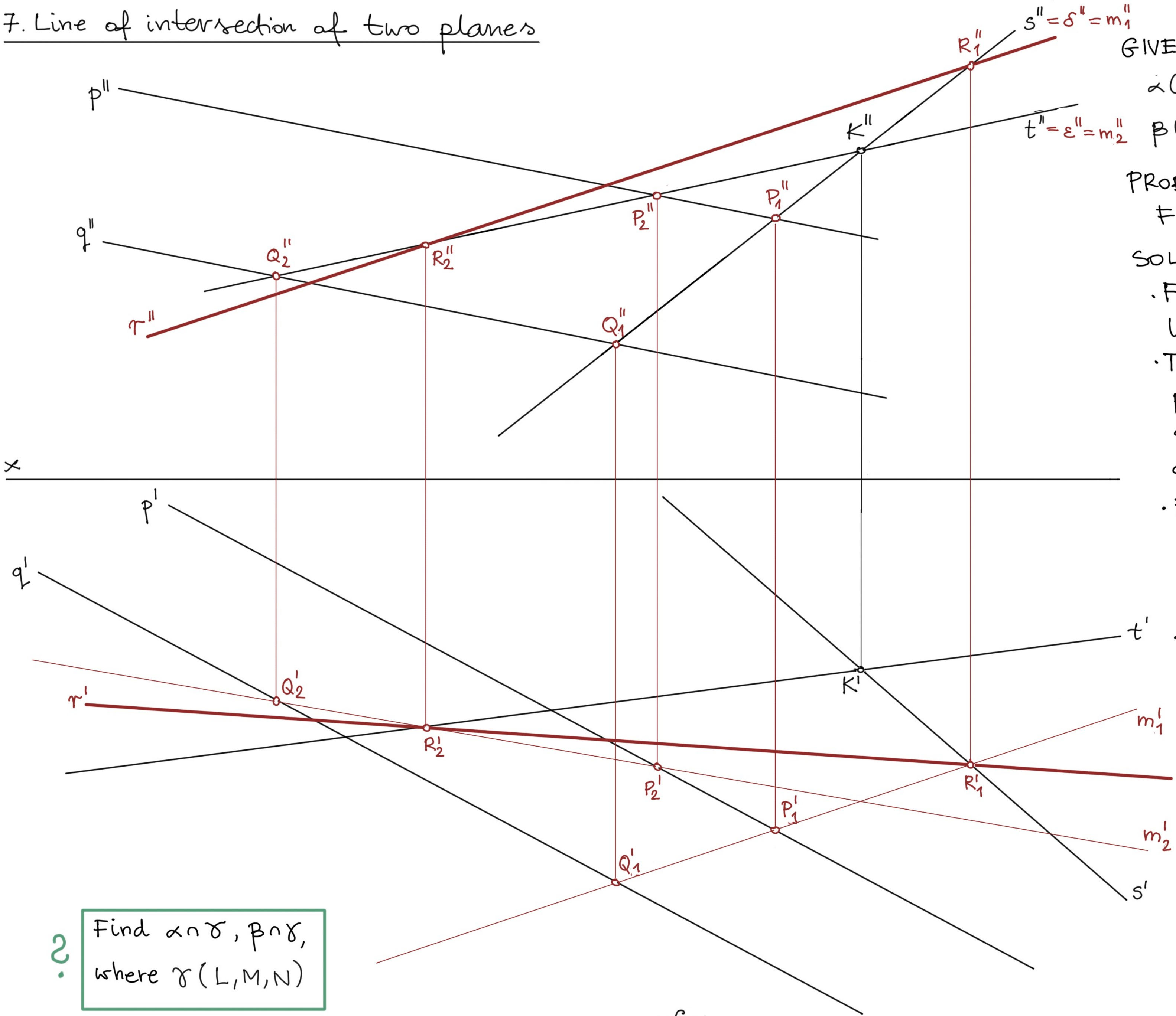
$P = s \cap t$, $P \in \beta$

From $s \in \alpha$ it follows that

$P \in \alpha$

? As above for $\alpha(p,q)$, where $p \parallel q$.

7. Line of intersection of two planes



GIVEN:

$\alpha(p,q)$, $p \parallel q$, $p \neq q$

$\beta(s,t)$, $K = s \cap t$

PROBLEM:

Find $r = \alpha \cap \beta$

SOLUTION:

- Follow the purple lines.
- Take δ, ε – auxiliary planes, $\{\delta, \varepsilon\} \perp \pi_2$

  $s \in \delta$, $t \in \varepsilon$

  $\delta' = \varepsilon' = \pi_1$
- Find $P_1 = p \cap \delta \Rightarrow P_1 \in \delta$

  $P_2 = p \cap \varepsilon \Rightarrow P_2 \in \varepsilon$

  $Q_1 = q \cap \delta \Rightarrow Q_1 \in \delta$

  $Q_2 = q \cap \varepsilon \Rightarrow Q_2 \in \varepsilon$
- Since

  $\{P_1, P_2, Q_1, Q_2\} \in \alpha$,

  then

  $m_1(P_1, Q_1) = \alpha \cap \delta$

  $m_2(P_2, Q_2) = \alpha \cap \varepsilon$
- Consequently

  $R_1 = s \cap \alpha = s \cap m_1$

  $R_1 \in s \in \beta$

  and

  $R_1 \in \alpha$
- Similarly,

  $R_2 = t \cap \alpha = t \cap m_2$

  $R_2 \in \beta$, $R_2 \in \alpha$

?

Find $\alpha \cap \gamma$, $\beta \cap \gamma$,

where $\gamma(L, M, N)$